EXAM II
CALCULUS AB
SECTION I   PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

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1. Let $f(x) = 4x^3 - 3x - 1$. An equation of the line tangent to $y = f(x)$ at $x = 2$ is

   (A) $y = 25x - 5$

   (B) $y = 45x + 65$

   (C) $y = 45x - 65$

   (D) $y = 65 - 45x$

   (E) $y = 65x - 45$

Ans
☐

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2. $\displaystyle\int_0^1 \sin \pi x \, dx =$

   (A) $\dfrac{2}{\pi}$ (B) $\dfrac{1}{\pi}$ (C) 0   (D) $-\dfrac{2}{\pi}$ (E) $-\dfrac{1}{\pi}$

Ans
☐

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3. $\displaystyle\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)=$

   (A) $\sin x$

   (B) $-\sin x$

   (C) $\cos x$

   (D) $-\cos x$

   (E)   does not exist

Ans ☐

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4.   On which of the following intervals, is the graph of the curve $y = x^5 - 5x^4 + 10x + 15$ concave up ?

   I. $x < 0$ II. $0 < x < 3$ III. $x > 3$

   (A)   I only

   (B)   II only

   (C)   III only

   (D)   I and II only

   (E)   II and III only

Ans ☐

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5.   The region bounded by the x-axis and the part of the graph of $y = \sin x$ between $x = 0$ and $x = \pi$ is separated into two regions by the line $x = k$. If the area of the region for $0 \le x \le k$ is one-third the area of the region for $k \le x \le \pi$, then $k =$

   (A) $\arcsin \dfrac{1}{3}$

   (B) $\arcsin \dfrac{1}{4}$

   (C) $\dfrac{\pi}{6}$

   (D) $\dfrac{\pi}{3}$

   (E) $\dfrac{\pi}{4}$

Ans ☐

6. A particle starts at time $t = 0$ and moves along a number line so that its position, at time $t \geq 0$, is given by $x(t) = (t-2)^3(t-6)$. The particle is moving to the right for

   (A) $0 < t < 5$

   (B) $2 < t < 6$

   (C) $t > 5$

   (D) $t \geq 0$

   (E) never

Ans ☐

7. If $\dfrac{dy}{dx} = \sec x$, then $y =$

   (A) $\ln|\cos x| + C$

   (B) $\sec x + \tan x + C$

   (C) $\ln|(\sec x)(\tan x)| + C$

   (D) $(\sec x)(\tan x) + C$

   (E) $\ln|\sec x + \tan x| + C$

Ans ☐

8. $\displaystyle\int_{\pi/4}^{\pi/3} \dfrac{\sec^2 x}{\tan x}\, dx =$

   (A) $\ln \sqrt{3}$ (B) $-\ln \sqrt{3}$ (C) $\ln \sqrt{2}$ (D) $\sqrt{3} - 1$ (E) $\ln \frac{\pi}{3} - \ln \frac{\pi}{4}$

Ans ☐

9.   What is $\displaystyle\lim_{x\to\infty} \frac{x^2 - 6}{2 + x - 3x^2}$?

    (A) −3   (B) $-\dfrac{1}{3}$ (C) $\dfrac{1}{3}$ (D) 2   (E) The limit does not exist.

Ans

10. $\displaystyle\int_0^2 \sqrt{x^2 - 4x + 4}\ dx$ is:

    (A) 1

    (B) −1

    (C) −2

    (D) 2

    (E) None of the above

Ans

11.   If $g(x) = \dfrac{x - 2}{x + 2}$, then $g'(2) =$

    (A) 1

    (B) −1

    (C) $\dfrac{1}{4}$

    (D) $-\dfrac{1}{4}$

    (E) 0

Ans

12. If $\dfrac{dy}{dx} = 2xy$ and if $y = 4$ when $x = 0$, then $y =$

(A) e^{x^2}

(B) $4e^{x^2}$

(C) $4 + e^{x^2}$

(D) $4 + 4e^{x^2}$

(E) $4 + 2e^{x^2}$

Ans

☐

13. The fourth derivative of $f(x) = (2x - 3)^4$ is

(A) $24\left(2^4\right)$

(B) $24\left(2^3\right)$

(C) $24(2x - 3)$

(D) $24\left(2^5\right)$

(E) 0

Ans

☐

14. If $\displaystyle\int_2^4 f(x)\, dx = 6$, then $\displaystyle\int_2^4 \left(f(x) + 3\right) dx =$

(A) 3

(B) 6

(C) 9

(D) 12

(E) 15

Ans

☐

15. If $\tan(x + y) = x$, then $\dfrac{dy}{dx} =$

(A) $\tan^2(x + y)$

(B) $\sec^2(x + y)$

(C) $\ln\left|\sec(x + y)\right|$

(D) $\sin^2(x + y) - 1$

(E) $\cos^2(x + y) - 1$

Ans

16. If $f(x) = e^{2x}$ and $g(x) = \ln x$, then the derivative of $y = f(g(x))$ at $x = e$ is

(A) e^2

(B) $2e^2$

(C) $2e$

(D) 2

(E) undefined

Ans

17. The area of the region bounded by the lines $x = 1$ and $y = 0$ and the curve $y = xe^{x^2}$ is

(A) $1 - e$

(B) $e - 1$

(C) $\dfrac{e - 1}{2}$

(D) $\dfrac{1 - e}{2}$

(E) $\dfrac{e}{2}$

Ans

18.   If $h(x) = (x^2 - 4)^{3/4} + 1$, then the value of $h'(2)$ is

   (A) 3

   (B) 2

   (C) 1

   (D) 0

   (E) does not exist

                                                                                    Ans

                                                                                    □

19.   The derivative of $\sqrt{x} - \dfrac{1}{x\sqrt[3]{x}}$ is

   (A) $\dfrac{1}{2}x^{-1/2} - x^{-4/3}$

   (B) $\dfrac{1}{2}x^{-1/2} + \dfrac{4}{3}x^{-7/3}$

   (C) $\dfrac{1}{2}x^{-1/2} - \dfrac{4}{3}x^{-1/3}$

   (D) $-\dfrac{1}{2}x^{-1/2} + \dfrac{4}{3}x^{-7/3}$

   (E) $-\dfrac{1}{2}x^{-1/2} - \dfrac{4}{3}x^{-1/3}$

                                                                                    Ans

                                                                                    □

20.   The function f is continuous at $x = 1$.

   If $f(x) = \begin{cases} \dfrac{\sqrt{x+3} - \sqrt{3x+1}}{x-1} & \text{for } x \neq 1 \\ \\ k & \text{for } x = 1 \end{cases}$ then $k =$

   (A) 0   (B) 1   (C) $\dfrac{1}{2}$ (D) $-\dfrac{1}{2}$ (E) none of the above

                                                                                    Ans

                                                                                    □

21. An equation of the normal to the graph of $f(x) = \dfrac{x}{2x - 3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

Ans
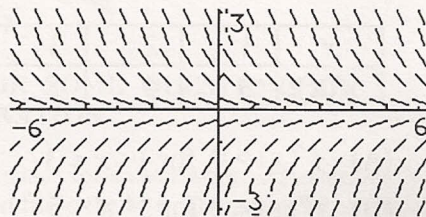
22. Let $f(x) = x \ln x$. The minimum value attained by f is

(A) $-\dfrac{1}{e}$
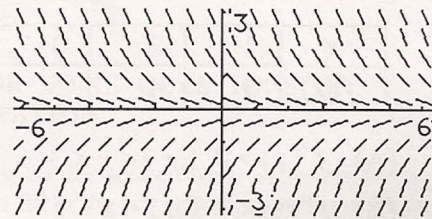
(B) 0

(C) $\dfrac{1}{e}$

(D) -1

(E) There is no minimum.

Ans

23. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$
     is given in the figure. The slope field corresponds to
     which of the following differential equations?

     (A) $\frac{dy}{dx} = x + y$

     (B) $\frac{dy}{dx} = y^2$

     (C) $\frac{dy}{dx} = -y$

     (D) $\frac{dy}{dx} = e^{-x}$

     (E) $\frac{dy}{dx} = 1 - \ln x$

     Ans
     ☐

24. The average value of $\sec^2 x$ over the interval $0 \le x \le \frac{\pi}{4}$ is

     (A) $\frac{\pi}{4}$ (B) $\frac{4}{\pi}$ (C) $\frac{\pi}{8}$ (D) 1   (E) none of the above

     Ans
     ☐

25. Suppose that g is a function that is defined for all real numbers. Which of the following
     conditions assures that g has an inverse function?

     (A) $g'(x) < 1$, for all x

     (B) $g'(x) > 1$, for all x

     (C) $g''(x) > 0$, for all x

     (D) $g''(x) < 0$, for all x

     (E) g is continuous.

     Ans
     ☐

21. An equation of the normal to the graph of $f(x) = \dfrac{x}{2x-3}$ at $(1, f(1))$ is

(A) $3x + y = 4$

(B) $3x + y = 2$

(C) $x - 3y = -2$

(D) $x - 3y = 4$

(E) $x + 3y = 2$

Ans

22. Let $f(x) = x \ln x$. The minimum value attained by f is

(A) $-\dfrac{1}{e}$

(B) 0

(C) $\dfrac{1}{e}$

(D) -1

(E) There is no minimum.

Ans

23. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. The slope field corresponds to which of the following differential equations?

(A) $\frac{dy}{dx} = x + y$

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(C) $\frac{dy}{dx} = -y$

(D) $\frac{dy}{dx} = e^{-x}$

(E) $\frac{dy}{dx} = 1 - \ln x$

Ans ☐

24. The average value of $\sec^2 x$ over the interval $0 \le x \le \frac{\pi}{4}$ is

(A) $\frac{\pi}{4}$ (B) $\frac{4}{\pi}$ (C) $\frac{\pi}{8}$ (D) 1   (E) none of the above

Ans ☐

25. Suppose that g is a function that is defined for all real numbers. Which of the following conditions assures that g has an inverse function?

(A) $g'(x) < 1$, for all x

(B) $g'(x) > 1$, for all x

(C) $g''(x) > 0$, for all x

(D) $g''(x) < 0$, for all x

(E) g is continuous.

Ans ☐

26. The function f is continuous and differentiable on the closed interval $[1, 5]$. The table below gives selected values of f on this interval. Which of the following statements must be TRUE?

x	1	2	3	4	5
$f(x)$	3	4	5	3	-2

(A) $f'(x) > 0$ for $1 < x < 3$

(B) $f''(x) < 0$ for $3 < x < 5$

(C) The maximum value of f on $[1, 5]$ must be 5.

(D) The minimum value of f on $[1, 5]$ must be -2.

(E) There exists a number c, $1 < c < 5$ for which $f(c) = 0$.

Ans

27. If the function G is defined for all real numbers by $G(x) = \int_0^{2x} \cos(t^2)\, dt$, then $G'(\sqrt{\pi}) =$

(A) 2   (B) 1   (C) 0   (D) -1 (E) -2

Ans

28. At time t a particle moving along the x-axis is at position x. The relationship between x and t is given by: $tx = x^2 + 8$. At $x = 2$ the velocity of the particle is

(A) 1

(B) 2

(C) 6

(D) -2

(E) -1

Ans

EXAM II
CALCULUS AB
SECTION I   PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1)   The <u>exact</u> numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1.   Which of the following functions have a derivative at $x = 0$?

   I. $y = \left| x^3 - 3x^2 \right|$

   II. $y = \sqrt{x^2 + .01} - \left| x - 1 \right|$

   III. $y = \dfrac{e^x}{\cos x}$

   (A) None   (B) II only   (C) III only   (D) II and III only   (E) I, II, III

Ans

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2.   Water is pumped into an empty tank at a rate of $r(t) = 20e^{0.02t}$ gallons per minute. Approximately how many gallons of water have been pumped into the tank in the first five minutes?

   (A) 20 gal

   (B) 22 gal

   (C) 85 gal

   (D) 105 gal

   (E) 150 gal

Ans

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3.   Consider the function $f(x) = \dfrac{6x}{a + x^3}$ for which $f'(0) = 3$. The value of a is

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

Ans
☐

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4.   Which of the following is true about the function f if $f(x) = \dfrac{(x-1)^2}{2x^2 - 5x + 3}$?

I. f is continuous at $x = 1$.

II. The graph of f has a vertical asymptote at $x = 1$.

III. The graph of f has a horizontal asymptote at $y = \dfrac{1}{2}$.

(A) I only   (B) II only   (C) III only   (D) II and III only   (E) I, II, III

Ans
☐

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5.   If $y = u + 2e^u$ and $u = 1 + \ln x$, find $\dfrac{dy}{dx}$ when $x = \dfrac{1}{e}$

(A) e (B) $2e$ (C) $3e$ (D) $\dfrac{2}{e}$ (E) $\dfrac{3}{e}$

Ans
☐

6. Sand is being dumped on a pile in such a way that it always forms a cone whose base radius is always 3 times its height. The function V whose graph is sketched in the figure gives the volume of the conical sand pile, $V(t)$, measured in cubic feet, after t minutes. $\left(V(t) = \frac{1}{3}\pi r^2 h\right)$ At what approximate rate is the radius of the base changing after 6 minutes?
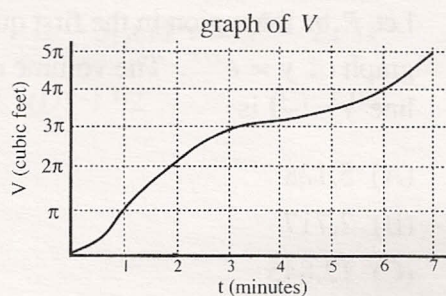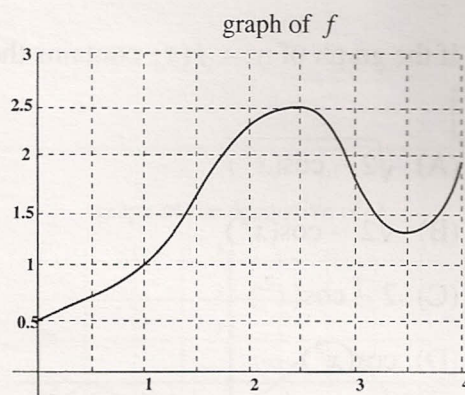
graph of V

(A) 0.22 ft/min   (B) 0.28 ft/min   (C) 0.34 ft/min   (D) 0.40 ft/min   (E) 0.46 ft/min

Ans

7. A graph of the function f is shown at the right. Which of the following statements are true?

graph of f

   I. $f(1) > f'(3)$

   II. $\displaystyle\int_1^2 f(x)\, dx > f'(3.5)$

   III. $\displaystyle\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} > \frac{f(2.5)-f(2)}{2.5-2}$

(A) I only   (B) II only   (C) I and II only   (D) II and III only   (E) I, II, III

Ans

8. Given: $5x^3 + 40 = \displaystyle\int_a^x f(t)\, dt$. The value of a is

   (A) –2
   (B) 2
   (C) 1
   (D) –1
   (E) 0

Ans

9. Let R be the region in the first quadrant enclosed by the lines $x = \ln 3$ and $y = 1$ and the graph of $y = e^{x/2}$. The volume of the solid generated when R is revolved about the line $y = -1$ is

    (A) 5.128

    (B) 7.717

    (C) 12.845

    (D) 15.482

    (E) 17.973

Ans

10. If the graph of $y = f(x)$ contains the point $(0, 1)$, and if $\dfrac{dy}{dx} = \dfrac{x \sin(x^2)}{y}$, then $f(x) =$

    (A) $\sqrt{2 - \cos(x^2)}$

    (B) $\sqrt{2} - \cos(x^2)$

    (C) $2 - \cos(x^2)$

    (D) $\cos(x^2)$

    (E) $\sqrt{2 - \cos x}$

Ans

11. If $y = \sin u$, $u = v - \dfrac{1}{v}$, and $v = \ln x$, then value of $\dfrac{dy}{dx}$ at $x = e$ is

    (A) 0

    (B) 1

    (C) $\dfrac{1}{e}$

    (D) $\dfrac{2}{e}$

    (E) $\cos e$

Ans

12. The area of the region bounded by the graphs of $y = 2 - |x - 3|$ and $y = x^2 - 2x$ is

   (A) 1.86   (B) 1.88   (C) 1.90   (D) 1.92   (E) 1.94

Ans

13. The figure shows the graph of f', the *derivative* of a function f. The domain of f is the interval $-4 \leq x \leq 4$. Which of the following are true about the graph of f?

graph of the derivative of f

  I. At the points where $x = -3$ and $x = 2$ there are horizontal tangents.

  II. At the point where $x = 1$ there is a relative minimum point.

  III. At the point where $x = -3$ there is an inflection point.

    (A) None   (B) II only   (C) III only   (D) II and III only   (E) I, II, III
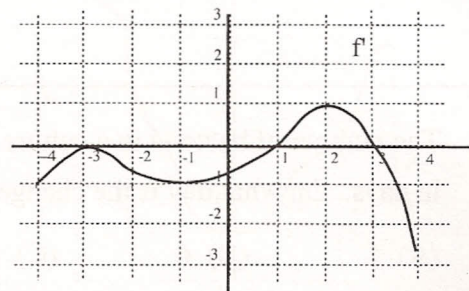
Ans

14. Consider the function $f(x) = (x^2 - 5)^3$ for all real numbers x. The number of inflection points for the graph of f is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

Ans

15. The number of bacteria in a culture is given by $N(t) = 200 \ln(t^2 + 36)$, where t is measured in days. On what day is the change in growth a maximum?

(A) 4   (B) 6   (C) 8   (D) 10   (E) 12

Ans

16. The acceleration of a particle at time t moving along the x-axis is given by: $a = 4e^{2t}$.
    At the instant when $t = 0$, the particle is at the point $x = 2$ moving with velocity $v = -2$.
    The position of the particle at $t = \dfrac{1}{2}$ is

    (A) $e - 3$ (B) $e - 2$ (C) $e - 1$ (D) e (E) $e + 1$

Ans

17. The graph of f', the derivative of a function f, is shown at the right.

    Which of the following statements are true about the function f?

    I. f is increasing on the interval $(-2, -1)$.
    II. f has an inflection point at $x = 0$.
    III. f is concave up on the interval $(-1, 0)$.

    the graph of f'
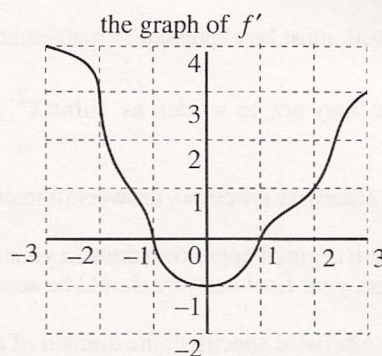
    (A) I only   (B) II only   (C) III only   (D) I and II only   (E) II and III only

Ans

EXAM II
CALCULUS AB
SECTION II, PART A
Time–45 minutes
Number of problems–3

**A graphing calculator is required for some problems or parts of problems.**

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.

- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.

- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.

- Justifications require that you give mathematical (noncalculator) reasons.

- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

- Your work must be expressed in mathematical notation rather than calculator syntax. For example,
$$\int_1^5 x^2 \, dx \text{ may not be written as } \text{fnInt}(X^2, X, 1, 5).$$

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.

- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

### THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 4 - x^2$ and $y = 1 + 2 \sin x$ as shown in the figure at the right.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Find the volume of the solid whose base is R and whose cross sections perpendicular to the x-axis are squares.
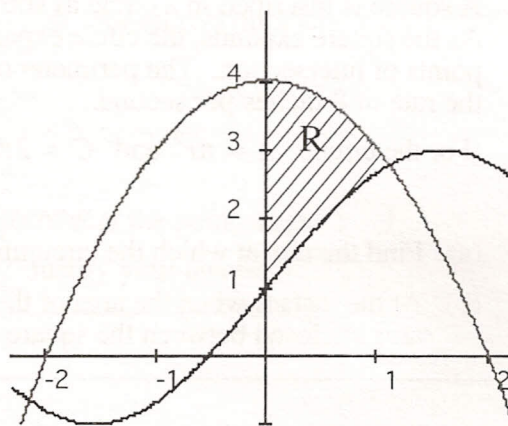
2. A square is inscribed in a circle as shown in the figure at the right. As the square expands, the circle expands to maintain the four points of intersection. The perimeter of the square is increasing at the rate of 8 inches per second.
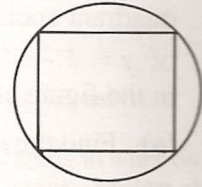
   (For the circle: $A = \pi r^2$ and $C = 2\pi r$.)

    (a) Find the rate at which the circumference of the circle is increasing.

    (b) At the instant when the area of the square is 16 square inches, find the rate at which the area enclosed between the square and the circle is increasing.

3. Suppose the function F is defined by $F(x) = \int_{1}^{\sqrt{x}} \frac{2t-1}{t+2} dt$ for all real numbers $x \geq 0$.

    (a) Evaluate $F(1)$.

    (b) Evaluate $F'(1)$

    (c) Find an equation for the tangent line to the graph of F at the point where $x = 1$.

    (d) On what intervals is the function F increasing? Justify your answer.

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK
ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

4.   Suppose that a population of bacteria grows according to the logistic equation
     $\frac{dP}{dt} = 2P(3-P)$, where P is the population measured in thousands and t is time measured in
     days. A slope field for this equation is given below.

   (a) Sketch the solution curve that passes through the point $(0, \frac{1}{10})$ and sketch the solution
        curve that passes through the point $(0, 4)$. Which solution has an inflection point?

   (b) Suppose the bacteria population began at day 0 with 1000 members, that is $P(0) = 1$.
        Find an equation of the line tangent to the solution curve $y = P(t)$ at the point $(0, 1)$.

   (c) Show that the function $P(t) = \frac{3}{1+2e^{-6t}}$ is a solution of the differential equation.

   (d) Show that the maximum growth rate of the bacteria occurs at $P = \frac{3}{2}$.
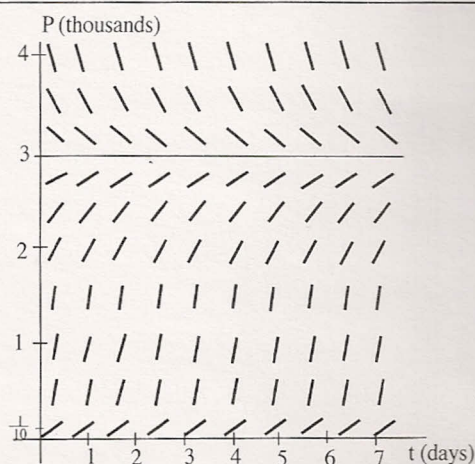
5. A function F is defined on the closed interval $-3 \le x \le 4$. The graph of F', the *derivative* of F, is shown at the right.

    graph of the derivative of F

    (a) On what intervals, if any, is F increasing. Justify your answer.

    (b) At what value of x does F attain its absolute maximum value on the closed interval $[-3, 4]$. Show the analysis that leads to your answer.

    (c) Find the interval(s) for which the graph of F is concave down.
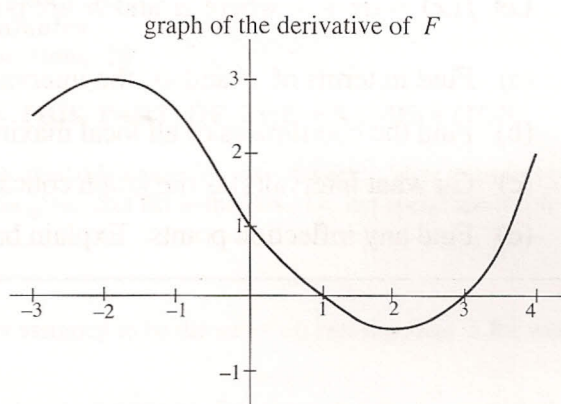
6.   Let $f(x) = ax + \dfrac{b}{x}$ where a and b are positive constants.

  (a) Find in terms of a and b, the intervals on which f is increasing.

  (b) Find the coordinates of all local maximum and minimum points.

  (c) On what interval(s) is the graph concave up?

  (d) Find any inflection points. Explain briefly.